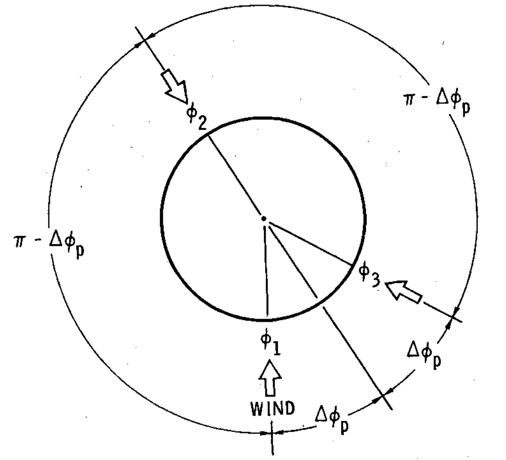

Fig. 3 Locus of minimum windward-meridian rotation rate.

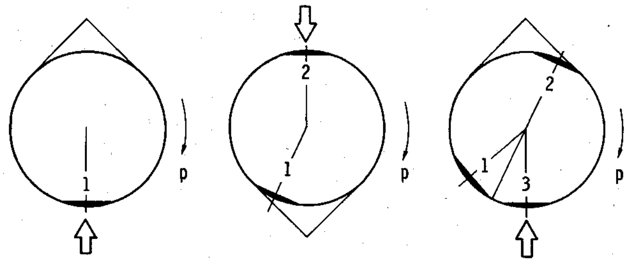

Fig. 4 Incipient asymmetry formation.

The behavior indicated in Figs. 3 and 4 suggests that a small shape change would occur with each pitch oscillation of the vehicle, so that the resulting trim moments would tend to cancel out. If the net trim remained sufficiently small, then the epicyclic motion would persist until it damped out from inherent pitch and normal force damping.[1] Because successive points of minimum $|\dot{\phi}|$ and presumed shape change occur on essentially opposite meridians (from the point of view of an observer fixed in space), the lift nonaveraging dispersion associated with equal increments of shape change should be minimal.[5,6]

It is emphasized that these conclusions are based on the assumption that very small "incipient" ablation or erosion asymmetries exist which do not influence the motion. A more rigorous treatment of the problem should include the effects of these shape asymmetries on the coupling between the angle of attack and windward-meridian rotation rate.

Acknowledgment

The author is grateful to R.F. Ross of R&D Associates for suggesting the locus of minimum windward-meridian rotation rate as a source of shape asymmetries that could influence the vehicle motion. This work was supported by the U.S. Air Force under Space Division Contract F040701-83-C-0084.

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Measurements of Despin and Yawing Moments Produced by a Viscous Liquid

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Nomenclature

I_A	= canister axial moment of inertia
I_0	= fixture moment of inertia (not including canister) about canister coning axis
I_T	= canister transverse moment of inertia
M_{F_Ω}	= bearing friction moment about canister coning axis
M_{F_ω}	= bearing friction moment about canister spin axis
M_{L_Ω}	= liquid fill induced yawing moment
M_{L_ω}	= liquid fill induced despin moment
XYZ	= aeroballistic axes system
θ	= canister coning angle
Ω	= canister coning rate
ω	= canister spin rate
$(\dot{\ })$	= first derivative with time

Introduction

SEVERE flight instabilities, characterized by a sharp increase in projectile yaw angle accompanied by an abrupt loss in spin rate have been experienced by spin-stabilized artillery projectiles having homogeneous, viscous liquid fills.[1] Experimental investigations have shown this flight instability to be produced by motion of the liquid fill and to have a maximum effect for a liquid kinematic viscosity on the order of 100 K cs.[2] Other experimental studies[3,4] indicate that this instability appears to be fundamentally different from the Stewartson type usually associated with projectiles having low viscosity liquid fills. Although several theoretical analyses employing different approaches[5-7] have been completed, additional experimental data are required to fully understand the source of the instability and support the evolution and validation of a comprehensive theoretical basis to describe the phenomena involved. This Note presents laboratory measurements of the liquid fill induced yawing and despin moments produced by the viscous liquid contained in a cylindrical canister undergoing simultaneous spinning and coning motion. The data provide experimental evidence of a direct relation between the destabilizing yawing moment and the despin moment produced by the viscous liquid fill.

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Experimental Approach

The basis of the experimental approach is that the motion of the spinning projectile can be represented by a fixed angle coning motion at the nutation frequency. The experimental measurements were obtained on a special laboratory test fixture which forces a projectile payload canister containing a viscous liquid to undergo the same simultaneous spinning and coning motion of the projectile in flight.[8] The viscous liquid fill experiences the same inertial effects it would have in flight, but in a known and controlled manner. This allows measurement of specific phenomena to be obtained.

The right circular cylindrical canister used in these tests is representative of a 155 mm artillery projectile payload compartment geometry and size. The canister had 100% fill (i.e., no void) of Dow Corning 200 silicone fluid having a specific density near unity and a kinematic viscosity of about 80 K cs. The physical values of the experimental elements are summarized in Table 1. Spin and coning rates employed in the experiments are representative of those possessed by a 155 mm artillery projectile in flight. All tests were conducted at a coning angle of 20 deg which is a large, but realistic, flight yaw angle. The angles, angular rates, moments of inertia, and moments are shown schematically in Fig. 1.

Viscous Liquid Induced Despin Moment

The initial experimental information obtained was the viscous liquid induced despin moment which is responsible for the rapid loss in projectile spin during flight. This moment can be measured easily and accurately on the fixture and is relatively simple to develop theoretically, compared to the yawing moment. The test procedure used to measure the despin moment was to allow a specific payload to despin while coning at a constant rate and angle. The net axial moment is determined by the product of the spin deceleration and the axial moment of inertia of the canister and viscous liquid payload. This net despin moment is due only to the spin bearing friction and the internal liquid effects. The bearing friction moment was evaluated separately by calibration runs with a solid canister. The difference between the net despin moment and the bearing friction moment is the despin moment induced by the viscous liquid fill. At a constant coning rate

$$M_{L_\omega} = I_A \dot{\omega} - M_{F_\omega} \tag{1}$$

Viscous Liquid Induced Yawing Moment

Experimental measurement of the liquid induced yawing moment is desirable because it is directly responsible for the yaw growth. It should be noted that the yawing moment which affects the nutational stability of the spinning projectile acts to yaw the canister out of the coning angle plane. The yawing moment is more difficult to determine than the despin moment due to its relatively small value compared to other yawing moments present on the test fixture and its combination with various external moments and inertial effects.

Table 1 Values of experimental terms

Canister/fixture characteristics	
Internal length, in.	19.62
Internal diameter, in.	4.36
Liquid total weight, lb	10.3
I_T, slug-ft^2	1.00
I_A, slug-ft^2	0.071
I_0, slug-ft^2	7.273
Characteristics, Dow Corning 200 silicone fluid	
Specific density, g/cm^3	0.98
Kinematic viscosity, K cs	80.0

The test procedure used to measure the liquid induced yawing moment was to have the liquid filled container simultaneously decone and despin at a fixed coning angle from a range of initial conditions. The liquid induced yawing moment (M_{L_Ω}) has a component directed along the fixture coning axis ($M_{L_\Omega}\sin\theta$). This component influences the coning deceleration and can be determined from the test fixture experimental data using the equation

$$M_{L_\Omega} = \frac{M_F^* - I_0^* \dot{\Omega}}{\sin\theta}$$

where

$$M_F^* = M_{F_\Omega} + M_{F_\omega}\cos\theta$$

and

$$I_0^* = I_0 + I_T \sin^2\theta$$

The net coning moment is the product of the coning deceleration and a term representing the fixture moment of inertia about the coning axis. The expression for the liquid induced yawing moment is only a function of the net coning moment and the effective coning bearing friction moment. The effective coning bearing friction moment was determined from calibration runs using a canister containing a solid fill having the same weight as the liquid fill. A negative sign of the liquid induced yawing moment would produce a destabilizing effect to the nutational motion of the projectile in flight.[9]

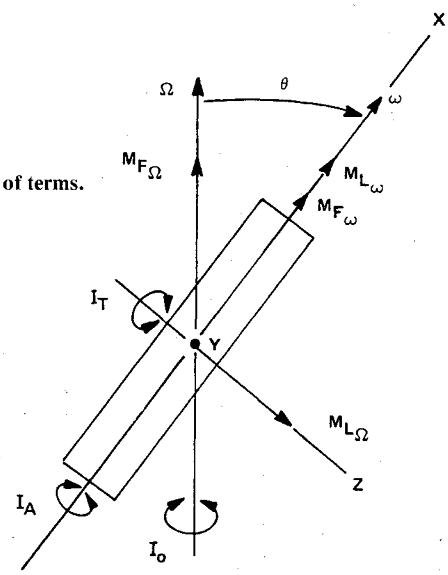

Fig. 1 Description of terms.

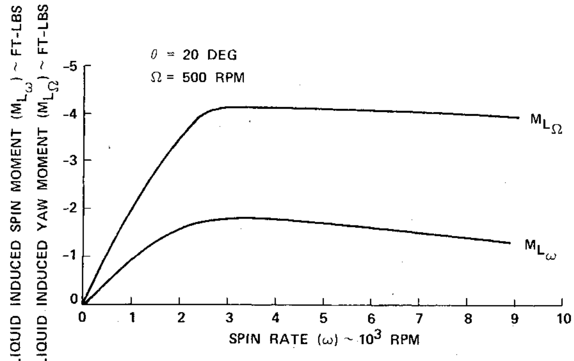

Fig. 2 Liquid induced despin and yawing moment vs spin rate.

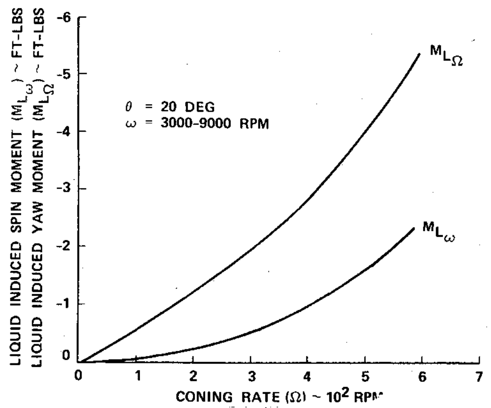

Fig. 3 Liquid induced despin and yawing moment vs coning rate.

Discussion of Results

The dependence of both the liquid induced yawing moment and spin moment with spin rate are similar as shown in Fig. 2. The spin moment was measured for $\dot{\omega} \cong -400$ rpm/s and $\dot{\Omega}=0$; the yaw moment measurement used $\dot{\omega} \cong -320$ rpm/s and $\dot{\Omega}=-6$ rpm/s. Note that both moments increase with spin rate up to about 2500 rpm and then remain fairly constant at higher spin rates. This general dependence was found to be similar for all coning rates. The relative independence of the moments on spin rate at the higher values of spin rate (representative of actual flight conditions) allows both moments to be plotted as a function of coning rate as contained in Fig. 3. This direct proportionality between the yawing moment and despin moment had previously been conjectured, but until now, never demonstrated experimentally.

It should be noted that these data apply specifically to a spin-stabilized artillery projectile. Accordingly, the detailed effect could be different for payload geometries, sizes, and rotational motion associated with other spinning flight vehicles.

The liquid viscosity and coning angle used in these experiments were selected to produce the largest despin and yawing moments possible on the test fixture. However, the results are representative of the phenomenon and provide heretofore unavailable quantitative data for comparison with theoretical predictions.

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Computerized Generation of Symbolic Equations of Motion for Spacecraft

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Introduction

For real-time simulations of spacecraft interfacing with hardware, the application of numerical formalisms is restricted due to the required computation time. Therefore, computer programs for the automatic generation of symbolic equations of motion for spacecraft have been developed, e.g., Macala[1] and Rosenthal and Sherman.[2] However, from an engineering point of view, it may be more efficient to extend an existing well-documented and thoroughly tested terrestrial multibody formalism to include spacecraft.

A formalism for the generation of symbolic equations of motion of terrestrial multibody systems have been presented in Ref. 3. The NEWEUL program based on this formalism has provided its high reliability for many technical applications, and can also be extended to orbiting spacecraft.[4] The necessary dynamical background is given herein.

Dynamics of Multibody Systems

In Eq. (25) of Ref. 3, the Newton-Euler equations of a system of p rigid bodies with f degrees of freedom are summarized as

$$\bar{M}(y,t)\ddot{y}+\bar{k}(y,\dot{y},t)=\bar{q}^e(y,\dot{y},t)+\bar{Q}(y,t)g \tag{1}$$

where y is the $f\times 1$ vector of generalized coordinates, and $\bar{M}$ is a $6p\times f$ inertia matrix given by

$$\bar{M}=\bar{\bar{M}}\bar{J} \tag{2}$$

Furthermore, the $6p\times 6p$ block-diagonal matrix of masses m_i and inertia tensors I_i is

$$\bar{\bar{M}}=\text{diag}\{m_1E,\ldots,m_pE,\quad I_1,\ldots,I_p\} \tag{3}$$

with the 3×3 identity matrix E, and the global $6p\times f$ Jacobian matrix

$$\bar{J}=[J_{T1}^T,\ldots,J_{Tp}^T,\quad J_{R1}^T,\ldots,J_{Rp}^T]^T \tag{4}$$

is composed of $3\times f$ Jacobian matrices J_{Ti} and J_{Ri} of translation and rotation. The $6p\times 1$ vectors $\bar{k}$ and $\bar{q}^e$ describe the gyroscopic and applied forces, respectively. The $6p\times q$ distribution matrix $\bar{Q}$ assigns the $q\times 1$ vector g of generalized constraint forces to each body of the system. Application of Lagrange's form of D'Alembert's principle leads to a premultiplication of Eq. (1) with $\bar{J}^T$ and results in the complete elimination of all constraint forces. Then, the equations of motion are given by the $f\times 1$ second-order vector differential equation

$$M(y,t)\ddot{y}+k(y,\dot{y},t)=q(y,\dot{y},t) \tag{5}$$

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